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Validation Over Basic Set Operations Of Internal Structure of MultiGranular Rough Sets

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Abstract: The concept of crisp sets was extended to the new set which was introduced by Pawlak namely rough sets. Some of the reasons for such extension are to undertake modelling, reasoning and other computing like modelling imprecise data. Many extensions have been made in various directions in order to improve the modelling capacity of the general rough sets. Among the several extensions of rough sets multigranulation rough set model is one among that and found high level practical usage since it deals with multiple granules. Granules with more than one equivalence relation enrich the detail level in the defined equivalence relation. Several fundamental properties of these types of rough sets have been studied. Pawlak introduced the types of rough sets in as an interesting characterization of rough sets by employing the ideas of lower and upper approximations of rough sets. There are two different ways of characterizing rough sets. The accuracy coefficient and the topological characterization introduced through the notion of types. As referred by Pawlak himself, in general rough sets by knowing the accuracy of a set, it is still difficult to tell exactly its topological structure and also the present knowledge about the topological structure of the set gives no information about its accuracy. Therefore in practical applications of rough sets it is better to combine both kinds of information about the borderline region, that is of the accuracy measure as well as the information about the topological classification of the set under consideration. Keeping this in mind, Tripathy and Mitra [9] have studied the types of rough sets by finding out the types of union and intersection of rough sets of different types. Later Raghavan et al have extended these results to the multigranular context in [12]. The database based validation of union of two multigranular rough sets was carried out by Raghavan in [16]. In this paper I provided other basic set operations of internal structure of multigranulation rough sets supported with database based validation.
Keywords: crisp sets, rough sets, multigranular rough sets, database, topological property.

Introduction

In general knowledge possessed by the human plays an essential role to classify the objects of a universe is the basic postulation of rough set theory. Classifications of a universe and equivalence relations defined on it together are recognized to be compatible ideas. For some of the mathematical reasons equivalence relations were considered by Pawlak to define rough sets. A rough set was represented by a pair of crisp sets, called the lower approximation and upper approximation. The lower approximation consists of only certain elements where as upper approximations comprise of all possible elements with respect to the available information in the form of granules which was defined over the equivalence relations. Several extensions have been made in different directions in order to improve the modelling capacity of the basic rough sets. From the point of view of granular computing, the classical rough set theory was researched by a single granulation. The basic rough set model has been extended to rough set model based on multigranulations (MGRS) in [10], where the set approximations are defined by using multiple equivalence relations on the universe. Several fundamental properties of these types of rough sets have been studied [10,11,17,18]. Pawlak introduced the types of rough sets in [3] as an interesting classification of rough sets by employing the ideas of lower and upper approximations of rough sets. There are two different ways to distinguish rough sets; the accuracy coefficient and the topological characterization introduced through the notion of types. As referred by Pawlak himself [8], in general rough sets by knowing the accuracy of a set, it is still unable to tell exactly its topological structure and also the present knowledge about the topological structure of the set gives no information about its accuracy. Therefore in practical applications of rough sets it is better to combine both kinds of information about the borderline region, that is of the accuracy measure as well as the information about the topological classification of the set under consideration.

Definitions

Definition: Let K= (U, **R**) be a knowledge base, **R** be a family of equivalence relations, X $\subseteq$ U and $R, S \in \mathbf{R}$. **T**he optimistic multi-granular lower approximation and optimistic multi-granular upper approximation of X with respect to R and S in U is defined as

$\underline{R+S}\,X = \{ x \mid [x]_R \subseteq X \text{ or } [x]_S \subseteq X\}$

and

$\overline{R+S}\,X = \sim(\underline{R+S}(\sim X))$.

Definition: Let K= (U, **R**) be a knowledge base, **R** be a family of equivalence relations, $X \subseteq U$ and $R, S \in \mathbf{R}$. **T**he pessimistic multi-granular lower approximation and pessimistic multi-granular upper approximation of X with respect to R and S in U is defined as

$\underline{R*S}\,X = \{ x \mid [x]_R \subseteq X \text{ and } [x]_S \subseteq X\}$

and $\overline{R*S}\,X = \sim(\underline{R*S}(\sim X))$.

Internal structure of basic rough sets

Topological property generally deals with the internal structures of sets. An interesting classification of rough sets was introduced by Pawlak, namely the topological characterization or classification of rough sets [8]. This topological characterization is found to be an additional one to the characterization of rough sets by means of numerical values in the form of accuracy coefficients. While differentiating the topological characterization and accuracy coefficient Pawlak expressed that "The accuracy coefficient expresses how large the boundary region of the set is, but says nothing about the structure of the boundary, whereas the topological classification of rough sets gives no information about the size of the boundary region but provides us with some insight as to how the boundary region is structured" [8]. In general, topological properties of sets deal with the internal structures of sets. The following four types were defined by the Pawlak.

There are four different kinds of rough sets. These are defined as follows:

Type 1: If $\underline{R}X \neq \phi$ and $\overline{R}X \neq U$ then X is roughly R-definable.

Type 2: If $\underline{R}X = \phi$ and $\overline{R}X \neq U$ then X is internally R-undefinable.

Type 3: If $\underline{R}X \neq \phi$ and $\overline{R}X = U$ then X is externally R-undefinable.

Type 4: If $\underline{R}X = \phi$ and $\overline{R}X = U$ then X is totally R-undefinable.

In terms of their types a study was recently made by Tripathy and Mitra [9] and obtained some interesting properties on intersection and union of rough sets of different types for single granular rough sets (Basic rough sets).

Internal structure of Multigranular Rough Sets

The topological properties of rough sets was introduced by Pawlak in terms of their types was recently studied by Tripathy et al to find the types of union and intersection of such sets and also complement of one such set for single granular rough sets. In this work these results are extended to the multigranulation context in [12] and as a result the following types of multigranulation rough sets based on topological view are obtained.

Type-1: If $\underline{R+S}X \neq \phi$ *and* $\overline{R+S}X \neq U$ then X is roughly R+S-definable.

Type-2: If $\underline{R+S}X = \phi$ *and* $\overline{R+S}X \neq U$ then X is internally R+S- definable.

Type-3: If $\underline{R+S}X \neq \phi$ *and* $\overline{R+S}X = U$ then X is externally R+S-definable.

Type-4: If $\underline{R+S}X = \phi$ *and* $\overline{R+S}X = U$ then X is totally R+S-definable.

Results

Intersection of two multigranular rough sets

The following example is used to show for the multigranular rough set where both X and Y are of Type 1 and its intersection gives either type 1 or type 2 in two cases. Let

EXAMPLE

Let

$U / P = \{\{e_1, e_7\}, \{e_2, e_3, e_4, e_5, e_6\}, \{e_8\}\}$

$U / Q = \{\{e_1, e_2\}, \{e_3, e_4, e_5\}, \{e_6, e_7, e_8\}\}$

CASE 1

$$X = \{e_1, e_2, e_6, e_8\}$$

$$\sim X = \{e_3, e_4, e_5, e_7\}$$

$$\underline{P+Q}(X) = \{e_1, e_2\} \neq \phi$$

$$\overline{P+Q}X = \sim(\underline{P+Q}(\sim X))$$
$$= \sim(e_3, e_4, e_5) = \{e_4, e_5, e_6, e_7, e_8)$$
$$\neq \bigcup$$

So X is of Type-1

$$Y = \{e_1, e_8\}$$

$$\sim Y = \{e_2, e_3, e_4, e_5, e_6, e_7\}$$
$$\underline{P+Q}Y = \{e_8\} \neq \phi$$
$$\overline{P+Q}Y = \sim(\underline{P+Q}(\sim Y))$$
$$= \sim(e_3, e_4, e_5) = \{e_1, e_2, e_6, e_7, e_8\}$$
$$\neq \bigcup$$

So Y is of Type-1.

$$X \bigcap Y = \{e_1, e_8\}$$

$$\sim(X \bigcap Y) = \{e_2, e_3, e_4, e_5, e_6, e_7\}$$
$$\underline{P+Q}(X \bigcap Y) = \{e_8\} \neq \phi$$
$$\overline{P+Q}(X \bigcap Y) = \sim(\underline{P+Q}(\sim X \bigcap Y))$$
$$= \sim(e_3, e_4, e_5, e_6)$$
$$= (e_1, e_2, e_7, e_8)$$
$$\neq \bigcup$$
$$so\ X \bigcap Y\ is\ of\ Type-1$$

Case 2

$$X = \{e_1, e_2, e_6, e_8\}$$

And from the previous case X is of Type-1.

$$Y = \{e_1, e_3, e_4, e_5\}$$

$$\sim Y = \{e_2, e_6, e_7, e_8\}$$

$$\underline{P+Q}(Y) = \{e_3, e_4, e_5\} \neq \phi$$

$$\overline{P+Q}(Y) = \sim(\underline{P+Q}(\sim Y))$$

$=\sim(e_6, e_7, e_8)$
$=\{e_1, e_2, e_3, e_4, e_5\}$
$\neq \bigcup$

So Y is of Type-1.

The intersection result of X and Y is given as

$X \bigcap Y = \{e_1\}$
$\sim(X \bigcap Y) = \{e_2, e_3, e_4, e_5, e_6, e_7, e_8\}$
$\underline{P+Q}(X \bigcap Y) = \phi$
$\overline{P+Q}(X \bigcap Y) = \sim \underline{P+Q}(\sim(X \bigcap Y))$
$=\sim(e_2, e_3, e_4, e_5, e_6, e_7, e_8)$
$=\{e_1\}$
$\neq \bigcup$
$so\ X \bigcap Y\ is\ of\ Type-2$

The following table summarizes the above discussed results in the tabular form.

	Type of Y with respect to P+Q				
		T-1	T-2	T-3	T-4
Type of X with respect to P+Q	T-1	T-1 / T-2	T-2	T-1/T-2	T-2
	T-2	T-2	T-2	T-2	T-2
	T-3	T-1 / T-2	T-2	T-1/T-2/T-3/T-4	T-2 / T-4
	T-4	T-2	T-2	T-2/T-4	T-2 / T-4

Table 4.1.1 Type of X$\bigcap$Y with respect to P+Q

Complementation in multigranular rough sets

If X is Type-3 and its complement x^c is Type-2. This is proved with the following example. Let

$U / P = \{\{e_1, e_7\}, \{e_2, e_3, e_4, e_5, e_6\}, \{e_8\}\}$

$U / Q = \{\{e_1, e_2\}, \{e_3, e_4, e_5\}, \{e_6, e_7, e_8\}\}$

$X = \{e_1, e_2, e_4, e_6, e_8\}$

$\sim X = \{e_3, e_5, e_7\}$

$\underline{P+Q}(X) = \{e_1, e_2\} \neq \phi$

$\overline{P+Q}(X) = \sim (\underline{P+Q}(X^C)) = \sim(\phi) = U$

So X is of Type - 3.

$X^{C} = \sim X = \{e_3, e_5, e_7\}$

$\underline{P+Q}(X^{C}) = \phi$

$\overline{P+Q}(X^{C}) = \sim (\underline{P+Q}((X^{C})^{C})$

$= \sim ((\underline{P+Q})X)) = \sim (e_1, e_2)$

$= \{e_3, e_4, e_5, e_6, e_7, e_8\}$

$\neq \bigcup$

$so\ X^{c}\ is\ of\ Type-2.$

If X is Type-1 and its complement x^{c} is Type-1. This is proved with the following example.
Let

$\bigcup / P = \{\{e_1, e_7\}, \{e_2, e_3, e_4, e_5, e_6\}, \{e_8\}\}$

$\bigcup / Q = \{\{e_1, e_2\}, \{e_3, e_4, e_5\}, \{e_6, e_7, e_8\}\}$

$X = \{e_1, e_2, e_3, e_4, e_5\}$

$X^{c} = \{e_6, e_7, e_8\}$

$\underline{P+Q}(X) = \{e_1, e_2, e_3, e_4, e_5\} \neq \phi$

$\overline{P+Q}X = \sim (\underline{P+Q}(X^{C}))$

$= \sim (e_6, e_7, e_8)$

$= \{e_1, e_2, e_3, e_4, e_5\}$

$\neq \bigcup$

$so\ X\ is\ of\ Type-1$

$X^{c} = \{e_6, e_7, e_8\}$

$\underline{P+Q}(X^{c}) = \{e_6, e_7, e_8\} \neq \phi$

$\overline{P+Q}(X^{C}) = \sim (\underline{P+Q}((X^{C})^{C})$

$= \sim ((\underline{P+Q}(X))$

$= \sim (e_1, e_2, e_3, e_4, e_5)$

$= \{e_6, e_7, e_8\}$

$\neq \bigcup$

$so\ X^{C}\ is\ of\ Type-1.$

The following table summarizes the above discussed results in the tabular form.

X	X^{C}
T-1	T-1
T-2	T-3
T-3	T-2
T-4	T-4

Table 4.1.2 Type of complement (X^{C}) with respect to P+Q

Validation

The following database table is used for validating the results.

Faculty Name	Division	Grade	Highest Degree	Native State
Sam	NW	AP	M.C.A	Tamil Nadu
Ram	IS	Pr	Ph.D	Andhra Pradesh
Shyam	SE	APJ	M.sc.,	Tamil Nadu
Peter	AI	ASP	Ph.D	Tamil Nadu
Roger	ES	P	Ph.D	Tamil Nadu
Albert	AI	APJ	M.Sc.,	Andhra Pradesh
Mishra	ES	APJ	M.Sc.,	Tamil Nadu
Hari	IS	SP	Ph.D	Orissa
John	SE	AP	M.C.A	West Bengal
Smith	NW	ASP	Ph.D	Orissa
Linz	AI	SP	Ph.D	Orissa
Keny	SE	P	Ph.D	Karnataka
Williams	ES	ASP	Ph.D	Tamil Nadu
Martin	IS	APJ	M.Sc.,	Karnataka
Jacob	NW	APJ	M.Sc.,	Karnataka
Lakman	SE	ASP	Ph.D	West Bengal
Sita	AI	AP	M.Tech	Karnataka
Fatima	ES	AP	M.Tech	West Bengal
Mukerjee	IS	SP	Ph.D	West Bengal
Preetha	SE	SP	Ph.D	Karnataka

TABLE 4.1.3.1 FACULTY DETAILS DATABASE TABLE

The database which is stated above is used for the validation. Here NW indicates networks, SE indicates Software engineering, AI indicates Artificial Intelligence, ES indicated Embedded Systems, IS indicated Information Systems.

Similarly AP indicates Assistant Professor, APJ indicates Assistant Professor (Junior), ASP indicates Associate Professor, SP indicates Senior Professor, Pr indicates Professor.
The universe and other equivalence relations are defined below.

$$\bigcup = \{sam, smith, jacob, shyam, john, keny, lakman, pretha, peter, albert, linz, sita, roger,$$
$$mishra, williams, fatima, ram, hari, martin, mukherjee\}$$

$$\bigcup / HighestDegree = \{\{Shyam, albert, mishra, martin, jacob\}, \{sam, john\}, \{sita, fatima\},$$
$$\{ram, peter, roger, hari, smith, keny, linz, williams, lakman, mukherjee, preetha\}$$

$$\bigcup / Nativestate = \{\{sam, shyam, roger, mishra, williams\}, \{ram, peter\}, \{hari, smith, linz\},$$
$$\{peter, john, lakman, fatima, mukherjee\}, \{keny, martin, jacob, sita, pretha\}\}$$

4.1.3.1 VALIDATION OF INTERSECTION

The following example is used to show that for one multigranular rough set of type-4 and the other is type-1 and its intersection is of type -2 Let
U / P be U / Highest Degree
U / Q be U / Native State
in the following example
X = {sam, sita,mishra,ram,hari,peter}

$$X \; U \sim X = \bigcup$$
$$\underline{P+Q}X = \phi$$
$$\overline{P+Q}X = \sim (\underline{P+Q}(\sim X)) = \sim (\phi)$$
$$= \bigcup$$
$$so\ X\ is\ of\ Type-4.$$
$$Y = \{hari, smith, linz\}$$
$$and\ Y \bigcup \sim Y = \bigcup$$
$$\underline{P+Q}Y = \{hari, smith, linz\} \neq \phi$$
$$\overline{P+Q}Y = \sim (\underline{P+Q}(\sim Y))$$
$$= \{hari, smith, linz\} \neq \bigcup$$
$$so\ Y\ is\ of\ Type-1.$$

$$X \bigcap Y = \{hari\}$$
$$\overline{P+Q}(X \bigcap Y) = \sim (\underline{P+Q}(\sim (X \bigcap Y))$$
$$= \{hari, smith, linz\}$$
$$\neq \bigcup$$
$$\underline{P+Q}(X \bigcap Y) = \phi$$
$$so\ X \bigcap Y\ is\ of\ Type-2$$

4.1.3.2 VALIDATION OF COMPLEMENTATION

If X is Type-4 and its complement X^c is Type-4. This is proved with the following example. Let
U / P be U / Highest Degree
U / Q be U / Native State
in the following example

$$X = \{sam, sita, mishra, ram, hari, peter\}$$
$$\underline{P+Q}X = \phi$$
$$\overline{P+Q}X = \sim(\underline{P+Q}(X^C))$$
$$= \sim(\phi)$$
$$= \bigcup$$
$$so\ X\ is\ of\ Type - 4.$$

$$X^C = \{shyam, albert, martin, jacob,$$
$$fatima, john, roger, smith, linz, keny, williams,$$
$$lakman, mukherjee, pretha\}$$
$$\underline{P+Q}(X^C) = \phi$$
$$\overline{P+Q}(X^C) = \sim(\underline{P+Q}(X^C)^C)$$
$$= \sim(\phi) = \bigcup$$
$$so\ X^C\ is\ of\ Type - 4$$

Conclusion

In this paper the results of internal structure of multigranulation rough sets particularly intersection and complementation of two multigranular rough sets is validated with a faculty database table. These results would be highly useful for further studies in approximation of classification and rule generation. This could be even extended to pessimistic multigranular rough sets and to be validated with a suitable database as a future work.

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